Algebra Primer: Math Refresher for College Physics

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Introduction

People encounter physics with different levels of mathematical strengths and backgrounds. A large percentage of students have forgotten much of their mathematical knowledge from secondary education. Physics requires students to be confident in manipulating equations and be familiar with certain mathematical identities and tricks. There's no negotiating this reality.

This document helps dust off old algebra skills and (perhaps) gain a few more.[1] It's designed to test the majority of the math you'll need for a typical algebra-based introductory physics courses at the university level. The goal is to help students feel more confident when tackling homework so they can focus on the physics instead of the math. If all the examples contained within this document make sense and you can **explain** the concepts to another person, you are well prepared. Explanation to another person is often invaluable when testing one's own understanding.

*****READ THIS TO SAVE TIME*****

Page 3 contains a quiz that identifies what skills you might need to work on. Each question references the section in this document that is being tested. If you get stuck on a problem simply look up the related section, re-learn the concept and move on. I recommend taking this quiz before reading the whole document.

Contents

Mathematical Survey ($\sim$ 25min)

This survey is meant to quickly test your algebra skills and identify any gaps in knowledge you have. You will need **all** the math covered in this survey at some point in most university-level physics courses. It should take about ~25 minutes if the test taker needs to look up very little information to solve the problem.

***Each question tests a concept found in this document and contains a reference to that section e.g. Q1 (§3.5) means the question is testing ideas from section 3.5 of this document.

Q1. Solve for x (§1.1-1.3)

$$3x - 2x - 7 = 1 - 5x$$

Q2. Simplify the expression (§1.4)

$$(x^3x^7)^2$$

Q3. Simplify (§1.5)

$$\frac{x^8}{x}$$

Q4. **True or False:** Answer True if all the equal signs are True, answer False if even one of them is False. (§1.6)

$$\frac{16(x+1)}{3} \stackrel{?}{=} (x+1)\frac{16}{3} \stackrel{?}{=} 16\frac{(x+1)}{3} \stackrel{?}{=} \frac{16x+1}{3}$$

Q5. Simplify (The * asterisk means multiplication) (§1.7)

$$\frac{2 * \frac{6}{7}}{\frac{3}{4}}$$

Q6. Simplify (§1.8)

$$\left(\frac{5}{4}\right)^3 * \left(\frac{125}{37}\right)^{-1}$$

Q7. Simplify. Here Δy is our variable. (§1.9)

$$\left(\frac{2\Delta y}{(\Delta y)^2}\right)^3 * \left(\frac{1}{(\Delta y)^2}\right)^{-1}$$

Q8. Simplify and solve for variable t in terms of variable v_0. (§1.10)

$$9t^2 - 2.5v_0 = -v_0 + 4.5t^2$$

Q9. Given the system of equations below, simplify and solve for variable v in terms of variables M_1, M_2 and scalar value G. Here m is a scalar and F is a variable. (§1.11)

$$F = m\frac{v^2}{r}$$

$$F = \frac{G(mM_1 + mM_2)}{r}$$

Q10. Simplify the expression into the form: (?) + (?) - (?) (§2.1)

$$\frac{93x + 62xy^3 - 279xz^{-2}}{31x^2}$$

Q11. Find at least one solution for x in terms of positive, non-zero constants b, c. (§2.2)

$$bx^2 + 6cx + 5bc = 0$$

Q12. Solve these equations for x **without** using substitution. (§2.3)

$$7x - 4y = 4 \tag{1}$$

$$50x + 8y = 24 \tag{2}$$

Q13. Two measurements of pressure are taken P_1, P_2. If the equation for pressure is: (§2.4)

$$P = m\frac{g}{A}$$

what is $\frac{P_1}{P_2}$ =? if $m_2 = \frac{1}{6}m_1$ and g, A are constants.

Q14. Show the intermediate steps to prove the equal signs are True: (§2.5)

$$\frac{1}{2(z-1)} - \frac{1}{2(z+1)} = \cdots = \frac{1}{(z-1)(z+1)}$$

Q15. Expand the following to simplify exponential: (§2.6)

$$(5x + 3y)^2$$

Q16. Solve for $|x|$ where $||$ means absolute value (§3.2)

$$3\left|2x^2\right| = 4\left|x^3\right|$$

Q17. True or False (§4.1)

If we solve for variable z in the expression

$$\log_c(z) = k$$

for constant c, we get

$$z = k^c$$

Q18. Solve for x (approximately) (§4.2-4.3)

$$e^x = 20.08554$$

Q19. Simplify/expand the following using logarithmic properties (§4.2-4.3)

$$\frac{\log(10xy)}{\log(x^3)}$$

Q20. Convert 273° to radians (§5.1)

Q21. True or False: (§5.2)

$$\sin\theta \stackrel{?}{=} \sin\left(\theta - \frac{9\pi}{2}\right)$$

Q22. What is the range of possible values of: (§5.3)

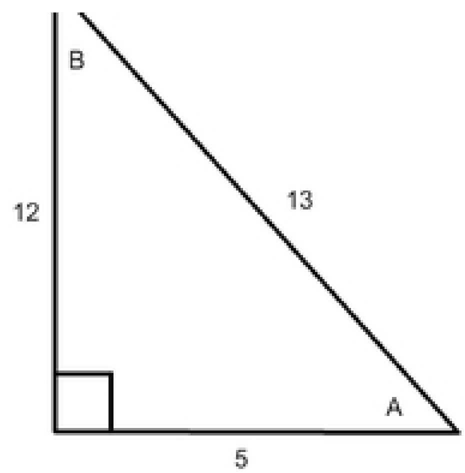
$$? \leq 9\pi\sin(\theta) \leq ?$$

Q23. True or False: (§5.3)

$$\sin(540°) \stackrel{?}{=} \cos(-270°)$$

Q24. True or False: if $\phi > 0$ is some constant, then the graph of $\tan(\theta - \phi)$ will look like $\tan(\theta)$ but shifted to the **right** by some amount. (§5.4)

Q25. For the following right triangle, find angle A (in radians) (§5.6)



Q26. Which of the follow expressions are True (if any)? (§6)

$$\frac{y}{x+z} \stackrel{?}{=} \frac{y}{x} + \frac{y}{z} \tag{1}$$

$$(x+a)^2 \stackrel{?}{=} x^2 + a^2 \tag{2}$$

Congratulations, You finished! Hopefully this jostled some old machinery in your brain! For the solutions to the Mathematical Survey, visit the link below

`https://rundus.github.io/HTML/documents/Primer_Solns.pdf`

1 Arithmetic operations on variables

For the following properties we use "a" and "b" to represent **positive** real numbers. The variable "x" (and sometimes "y") will be used in the properties definition. The examples problems can have any variable just like in physics.

1.1 Adding similar variables

Property:

$$ax + bx = (a + b)x \tag{1}$$

$\boxed{\text{Examples:}}$

$$19y + 5y = (19 + 5)y = 24y$$
$$3x + 2x = (3 + 2)x = 5x$$

1.2 Subtracting similar variables

Property:

$$(-a)x + bx = bx - ax = (b - a)x = (-a + b)x \tag{2}$$

$\boxed{\text{Examples:}}$

$$-5x + 3x = (-5 + 3)x = -2x$$
$$7z - 4z = (7 - 4)z = 3z$$
$$-9y - 18y = (-9 - 18)y = -27y$$

1.3 Multiplying numbers by variables

Property:

$$a * (bx) = b * (ax) = (a * b)x \tag{3}$$

(* means multiplication)

$\boxed{\text{Examples:}}$

$$4 * (7x) = (4 * 7)x = 28x$$
$$-8 * (2y) = 2 * (-8y) = (2 * -8)y = -16y$$

1.4 Exponents with variables

Properties:

$$x^a x^b = x^{a+b} \tag{4}$$

$$(x^a)^b = (x^b)^a = x^{a*b} = x^{b*a} = x^{ab} \tag{5}$$

$\boxed{\text{Examples:}}$

$$z^2 z^3 = z^{2+5} = z^5$$

$$(x^4)^2 = x^{4*2} = x^8$$

1.5 Dividing similar variables

Properties:

$$x^{-a} = \frac{1}{x^a} \tag{6}$$

$$\frac{x^a}{x^b} = x^a x^{-b} = x^{a-b} \tag{7}$$

$\boxed{\text{Examples:}}$

$$x^{-5} = \frac{1}{x^5}$$

$$\frac{y^5}{y^4} = y^5 y^{-4} = y^{5-4} = y^1 = y$$

1.6 Fraction rearranging

Property:

$$\frac{a * x}{b} = x\left(\frac{a}{b}\right) = \left(\frac{a}{b}\right)x \tag{8}$$

$\boxed{\text{Examples}}$

$$\frac{5x}{9} = x\left(\frac{5}{9}\right) = \left(\frac{5}{9}\right)x$$

1.7 Fractions divided by fractions

Property for real numbers "c" and "d" we have

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{a}{b}\right) * \left(\frac{d}{c}\right) = \frac{a * d}{b * c} = \frac{ad}{bc} \tag{9}$$

$\boxed{\text{Examples}}$

$$\frac{\left(\frac{5}{6}\right)}{7} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{7}{1}\right)} = \left(\frac{5}{6}\right) * \left(\frac{1}{7}\right) = \frac{5 * 1}{6 * 7} = \frac{5}{42}$$

$$\frac{\frac{9}{2}}{\frac{1}{7}} = \left(\frac{9}{2}\right) * \left(\frac{1}{7}\right) = \frac{9 * 7}{2 * 1} = \frac{63}{2}$$

$$\frac{5}{\frac{2}{3}} = 5 * \frac{3}{2} = \frac{15}{2}$$

1.8 Fractions and exponents

Properties

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \text{(for some number "n")} \tag{10}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a} \tag{11}$$

$\boxed{\text{Examples}}$

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

$$\left(\frac{2}{3}\right) * \left(\frac{2}{3}\right)^{-1} = \frac{2}{3} * \frac{3}{2} = \frac{2 * 3}{3 * 2} = 1$$

1.9 Fraction arithmetic using variables

We can combine all the above properties and relationships in order to solve a problems like these

$\boxed{\text{Examples:}}$

$$\frac{\frac{4x}{5}}{\frac{2x^2}{6}} = \left(\frac{4x}{5}\right)\left(\frac{6}{2x^2}\right) = \frac{4x * 6}{5 * 2x^2} = \frac{24x}{10x^2} = \left(\frac{24}{10x}\right) = \frac{24}{10}\frac{1}{x} = \frac{12}{5}\frac{1}{x}$$

$$\frac{\frac{1}{3}}{(\frac{2}{x})^2} = \frac{\frac{1}{3}}{\frac{2^2}{x^2}} = \frac{1}{3} * \frac{x^2}{4} = \frac{x^2}{12} = \frac{1}{12}x^2$$

1.10 Arithmetic for more than one variable

All of the above properties also apply for when we have multiple variables BUT we need to be careful as **they only apply to variables of the same type**.

$\boxed{\text{Examples:}}$

$$x + y = y + x$$
$$2x + (x + y) - 2y = 3x - y$$
$$(x + 2y)^2 = (x + 2y)(x + 2y) = x^2 + 2yx + 2yx + 4y^2 = x^2 + 4xy + 4y^2$$

1.11 Distributive Law

The distributive property is exceeding useful as it shows we can both distribute and reverse distribute (factoring out).

$$a(x + y) = ax + by = (x + y)a \tag{12}$$
$$az + bx = a(z + x) \tag{13}$$

$\boxed{\text{Examples:}}$

$$5(x + z) = 5x + 5z = (x + z)5$$
$$25x + 25z = 25(x + z)$$
$$E = mgh + \frac{1}{2}mv^2 = m(gh + \frac{1}{2}v^2) = \frac{1}{2}m(2gh + v^2)$$

This last equation is the conservation of energy is physics. The final expression on the right is easier to compute because we only have to plug "m" into our calculator once.

2 Tricks and Techniques of Equation Manipulation

2.1 Distributive "Trick"

$$ay + bz = a\left(\frac{a}{a}y + \frac{b}{a}z\right) = a\left(y + \frac{b}{a}z\right) = b\left(\frac{a}{b}y + z\right) \tag{14}$$

When is it used: This little trick is very helpful for cancelling out variables in physics equations or mostly can be used to clean up equations to make them look nice.

Examples:

$$35y + 5x = 5(\frac{35}{5}y + \frac{5}{5}x) = 5(7y + x)$$

$$\frac{100xz + 425yz}{25z} = \frac{25z(4x + 17y)}{25z} = \frac{25z}{25z}(4x + 17y) = (4x + 17y)$$

2.2 The quadratic formula

When is it used: The quadratic formula is just the solution to a very particular equation:

$$ax^2 + bx + c = 0$$

where a,b,c are just numbers and "x" is the variable (which can be **any** variable but we choose x arbitrarily). This has a solution that most people have seen:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The formula gives us 2 answers and as would-be physicists we usually choose the answer that makes physical sense (for example we don't accept negative values for the time variable).

Example:

Lets use physics equation

$$\frac{g}{2}t^2 + v_0 t - \Delta x = 0$$

that relates the **variable** time (t), and **constants**: initial velocity (v_0), acceleration (g) and position (Δx).

We can apply the quadratic formula with a= g/2, b = v_0 and c=$-\Delta x$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4(g/2)(-\Delta x)}}{2(g/2)}$$

Which we can clean up a little

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2g(\Delta x)}}{g}$$

2.3 Adding and subtracting equations

When is it used: Adding and subtracting whole equations is used to eliminate variables from equations in order to isolate one in particular. This trick stresses what is meant by an "=" sign.
Suppose we had

$$3x + 65y = 10$$
$$-3x + 35y = 2$$

Then we can do:

$$10 + 2 = (3x + 65y) + 2 = (3x + 65y) + (-3x + 35y) = 65y + 35y = 100y$$

So we see y= 1/10 and we could solve then for x in either of the equations we started with. But the trick doesn't end there! We could **add a multiple of one equation to another** and do the same thing. Suppose we had:

$$15x + y = 3$$
$$3x - 4y = -12$$

I want to eliminate x somehow, so I'll take the second equation, multiply it **all** by -5 then add it to the first equation:

$$(15x + y) - 5(3x - 4y) = y + 20y = 21y = 3 - (-5) * 12 = 63$$

So y=3 here from 21y = 63 in the equal signs.

(*** adding a negative equation, as in the second example, is equivalent to subtracting two equations)

2.4 Dividing equations

When is it used: Just like the above trick, the equal sign allows us to divide (or multiply) equations. This is especially useful in physics when we want to **compare** the relative strength of two variables. It can be used algebraically to simplify expressions

$\boxed{\text{Examples:}}$

A very common physics problem you will see on exams is something like:

Q: For the equation $E = mc^2$, we take two samples, giving us E_1, E_2, m_1, m_2. If we know $m_1 = \frac{1}{2}m_2$, what is $E_2 =$? (write your answer in terms of E_1)

So we can write out the two cases

$$E_1 = m_1c^2$$

$$E_2 = m_2c^2$$

Now we take a ratio to remove c^2

$$\frac{E_2}{E_1} = \frac{m_2c^2}{m_1c^2} = \frac{m_2}{m_1} = \frac{m_2}{(\frac{1}{2}m_2)} = \frac{1}{\frac{1}{2}} = 2$$

So by multiplying everything by E_1 we have

$$E_2 = 2E_1$$

In the world of college physics, **the majority of problem that asks for a one variable in terms of another can be solved by taking the ratio of the setup equations**.

2.5 Simplifying Algebraic Fractions

When is it used: When variables are in fractions we can reduce our expression by "distributing" the denominator and **only** the denominator. The numerator does not have the following property. We can also use our knowledge of fractions to find common denominators between fractions to reduce expressions.

Properties:

$$\frac{a}{c+d}+\frac{b}{c+d}=\frac{a+b}{c+d} \tag{15}$$

$\boxed{\text{Examples:}}$

$$\frac{(x+1)+(x+2)}{(x+1)}=\frac{x+1}{x+1}+\frac{x+2}{x+1}=1+\frac{x+2}{x+1}$$

$$\frac{4}{x-2}+\frac{3x+6}{(x-2)^2}=\frac{4(x-2)}{(x-2)^2}+\frac{3x+6}{(x-2)^2}=\frac{(4x-8)+(3x+6)}{(x-2)^2}=\frac{x-2}{(x-2)^2}=\frac{1}{x-2}$$

(See section 6 on "Common Algebra Errors to Avoid" to see the error in the numerator)

2.6 Expanding Squared/Cubed Expressions

When is it used: The technique of expanding expressions that are raised to powers is exceedingly useful for simplifying expressions.

Properties:

$$(a+b)^2=a^2+2ab+b^2 \tag{16}$$

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3 \tag{17}$$

One method of doing this is **The Punnett Square Method**, example below:

$\boxed{\text{Examples:}}$

(1) Expand

$$(6x + 2y)^2$$

We create our square and multiply the elements:

	6x	2y
6x	6x * 6x	2y * 6x
2y	6x*2y	2y*2y

We are left with

	6x	2y
6x	$36x^2$	12xy
2y	12xy	$4y^2$

We then add up all the elements to get our answer

$$(6x + 2y)^2 = 36x^2 + 12xy + 12xy + 4y^2 = \boxed{36\text{x}^2 + 24xy + 4y^2}$$

The same process can be applied to $(a + b + c)^2$ but you create a 3x3 Punnett square instead of a 2x2 and so on. Similarly, we can do the process for $(a+b)^3 = (a+b)^2(a+b)$ where we make 2 Punnett Squares, one for expanding $(a + b)^2$ and another for $(a^2 + 2ab + b^2)(a + b)$.

3 Ordering & Absolute Values

3.1 Ordering

- $<$ (less than)
- $>$ (greater than)
- $\leq$ (less than or equal to)
- $\geq$ (greater than or equal to)

3.2 Absolute Values

Properties:

$$|a| \geq 0 \tag{18}$$

$$|-a| = |a| \tag{19}$$

$$|ab| = |a||b| \tag{20}$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|} \tag{21}$$

$$|a+b| \leq |a| + |b| \quad \text{(Triangle Inequality)} \tag{22}$$

***The Triangle Inequality is used in physics when talking about the lengths of vectors.

$\boxed{\text{Examples:}}$

$$|5x| = |-5x| \geq 0$$

$$|-5yx| = |(-5y)x| = |(-5)y||x| = |-5||y||x| = 5|y||x|$$

$$\left|\frac{x-1}{-y}\right| = \frac{|x-1|}{|-y|} = \frac{|x-1|}{|y|} \leq \frac{|x|+|-1|}{|y|} = \frac{|x|+1}{|y|}$$

4 Logarithms

Fun fact: The invention of the Logarithm wasn't until the 1600's and revolutionized calculations in Astronomy. Before, people had to do long calculations by hand which could take decades! This invention gave astronomers enough time in their lives to actually do astronomy.

4.1 Definitions

The **general** logarithm is defined as

$$\log_a(x) = b$$

Where we read this in English as: log base "a" of "x" equals "b".

The logarithm was developed as an **inverse** function to the equation:

$$a^b = x$$

So from this we can interpret the logarithm as asking the following question: **what power do I need to raise "a" to in order to get "x"?**

$\boxed{\text{Example:}}$

$$2^3 = 8 \quad \& \quad \log_2(8) = 3$$

$$x =?\text{ for } (5.5)^x = 4 \quad \Longrightarrow \quad x = \log_{5.5}(4) = 0.813\ldots$$

4.2 Special Logarithms

In higher level mathematics it becomes clear to there are **special** numbers to choose for the base in the equation:

$$a^b = x$$

Choosing a=10 and a = e = 2.71828..., where e is Euler's number turns out to be really useful for a menagerie of advanced reasons. We give these logarithms special names:

- $\log_{10}(x) := \log(x)$ (called "log")
- $\log_e(x) := \ln(x)$ (called "The Natural Log")

4.3 Properties:

The following properties work for logarithms of **ANY** base.

$$\log_a(a) = 1 \tag{23}$$

$$\log_a(1) = 0 \tag{24}$$

$$\log_a(x^b) = b * \log_a(x) \tag{25}$$

$$\log_a(xy) = \log_a(x) + \log_a(y) \tag{26}$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y) \tag{27}$$

Examples:

$$\log_6(6) = 1$$

$$\log_{10}(1) = \log(1) = 0$$

$$\log_e(x^4) = \ln\big(x^4\big) = 4\ln(x)$$

$$\log(4yx) = \log((4y)x) = \log(4y) + \log(x)$$

$$\log_7(\frac{7}{5}) = \log_7(7) - \log_7(5) = 1 - \log_7(5)$$

5 Trigonometry

5.1 Radians Degrees conversion

There's two units physicists measure angles in: radians and degrees. Each circle has a total of $360°$ or 2π radians. Thus the conversion between the two is:

$$360° = 2\pi \text{ rad}$$

or

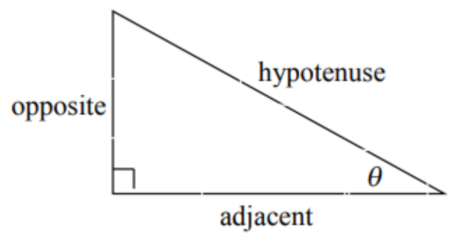
$$\frac{2\pi}{360°} = \frac{\pi}{180°}$$

$\boxed{\text{Examples:}}$

$$127^\circ * \frac{\pi}{180^\circ} \approx 2.22\text{rad}$$

$$\frac{7\pi}{5} * \frac{180^\circ}{\pi} = 252^\circ$$

5.2 Trigonometric Functions



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

The trigonometric functions are **periodic**. They repeat the same values after a certain θ value. That value is 360° or 2π radians.

$$\sin(\theta \pm 360^\circ) = \sin(\theta) \quad (\theta \text{ in degrees})$$
$$\sin(\theta \pm 2\pi) = \sin(\theta) \quad (\theta \text{ in radians})$$

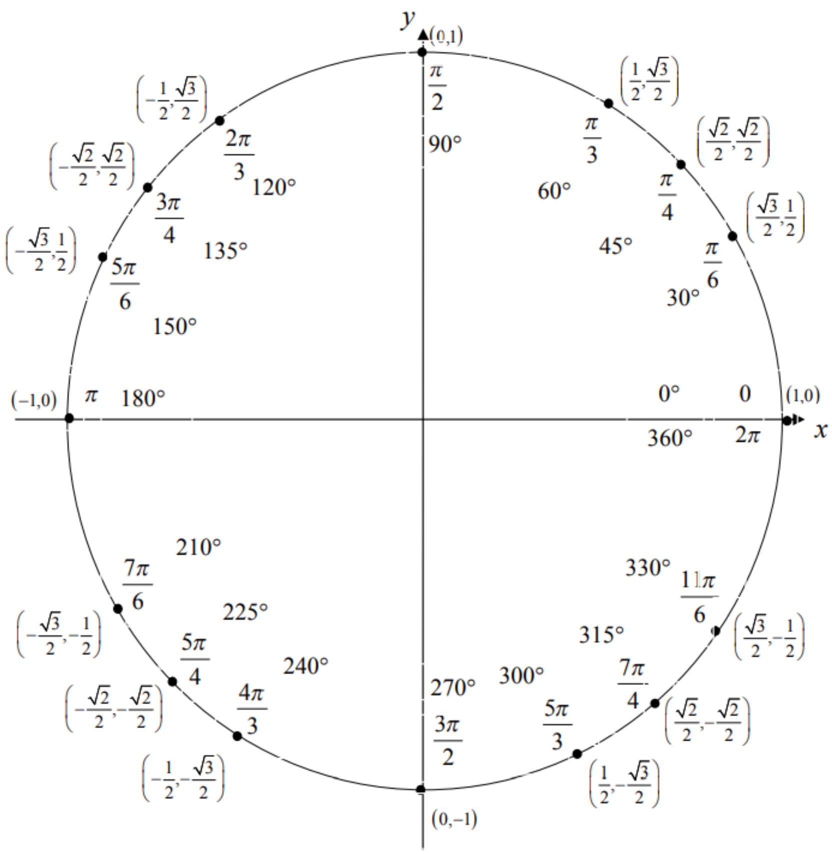
*** The same is also true for $\cos\theta$.

$$\tan(\theta \pm 180^\circ) = \tan(\theta) \quad (\theta \text{ in degrees})$$
$$\tan(\theta \pm \pi) = \tan(\theta) \quad (\theta \text{ in radians})$$

We can also do any whole multiple of these:

$$\sin(\theta + 2\pi n) = \sin(\theta), \quad n = 0, \pm 1, \pm 2 \ldots$$
$$\tan(\theta + \pi n) = \tan(\theta) \quad n = 0, \pm 1, \pm 2 \ldots$$

5.3 The Unit Circle



Memorizing the whole unit circle is **not** usually something you have to do, **but** knowing a couple of properties is necessary. Those are:

Properties:

$$-1 \leq \cos\theta \leq 1 \tag{28}$$

$$-1 \leq \sin\theta \leq 1 \tag{29}$$

$$-\infty < \tan\theta < \infty \tag{30}$$

$$-A \leq A * \sin(\theta) \leq A \tag{31}$$

$$\sin(0^\circ) = \sin(180^\circ) = 0 \tag{32}$$

$$\cos(90^\circ) = \sin(270^\circ) = 0 \tag{33}$$

5.4 Phase Angle

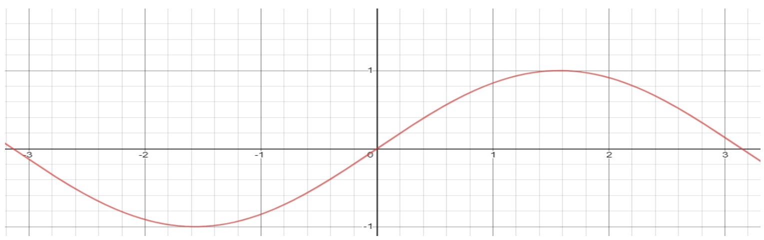
Phase Angle

Sometimes when using $\cos\theta$ or $\sin\theta$ it's often useful to know what happens to the graph of these functions when we add values to θ inside the function. In the case of studying waves in physics we might consider expressions like:

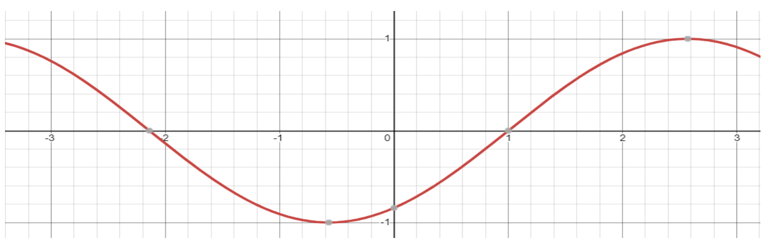
$$\cos(\theta \pm \phi) \quad \& \quad \sin(\theta \pm \phi)$$

Where ϕ is some new constant we add or take away from every θ value we insert. For example we consider the $\sin\theta$ function:

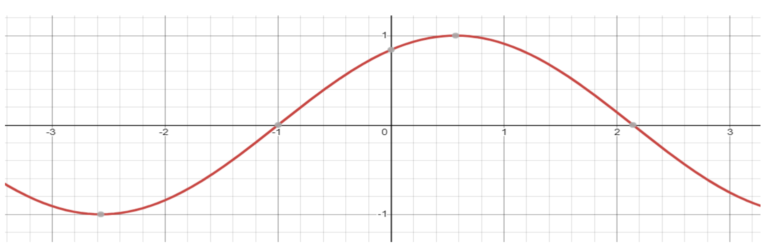
$\sin\theta$ (Normal)



$\sin(\theta + 1)$ (graph is shifted **left**)



$\sin(\theta - 1)$ (graph is shifted **right**)



This property is true in general so for cosine, sine or tangent:

$$\theta \pm \phi = \begin{cases} -\phi & \text{Shift Right} \\ +\phi & \text{Shift Left} \end{cases}$$

5.5 Useful Trig Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{34}$$

$$\sin^2\theta + \cos^2\theta = 1 \tag{35}$$

$$\sin(-\theta) = -\sin(\theta) \tag{36}$$

$$\cos(-\theta) = \cos(\theta) \tag{37}$$

5.6 Inverse Trig Functions

There are functions that can "undo" a trigonometric function. We call these "arcsine", "arccosine" and "arctangent" and one way they're written is:

$$\sin^{-1}(x)$$
$$\cos^{-1}(x)$$
$$\tan^{-1}(x)$$

Which should not be confused with:

$$\sin^{-1}(x) \neq \frac{1}{\sin(x)}$$

Instead:

$$(\sin(x))^{-1} = \frac{1}{\sin(x)}$$

Properties:

$$\sin^{-1}(\sin(\theta)) = \theta \tag{38}$$

$$\cos^{-1}(\cos(\theta)) = \theta \tag{39}$$

$$\tan^{-1}(\tan(\theta)) = \theta \tag{40}$$

$\boxed{\text{Example}}$ Solve for θ in $\tan[\sin\theta + 1] = 5$

$$\begin{aligned}
\tan[\sin(\theta) + 1] &= 5 \\
\implies \quad \tan^{-1}(\tan[\sin(\theta) + 1]) &= \tan^{-1}(5) \\
\implies \quad \sin(\theta) + 1 &= \tan^{-1}(5) \\
\implies \quad \sin(\theta) &= \tan^{-1}(5) - 1 \\
\implies \quad \sin^{-1}(\sin(\theta)) &= \sin^{-1}[\tan^{-1}(5) - 1] \\
\implies \quad \theta &= \sin^{-1}[\tan^{-1}(5) - 1]
\end{aligned}$$

6 Common Algebra Errors to Avoid & Further Algebraic Tools

There's a slew of things that can go wrong while doing algebra (I have lots of personal experience here), but there's common mistakes you can avoid by being careful. Here I list the ones I see the most when teaching:

$$\begin{aligned}
(x^2)^4 &\neq x^6 && \text{(See Section 1.4)} \\
-a(x-1) &\neq -ax - a && \text{(See Section 1.11)} \\
\frac{y}{x+z} &\neq \frac{y}{x} + \frac{y}{z} && \text{(See Section 1.5)} \\
\frac{1}{x^2+x^3} &\neq \frac{1}{x^2} + \frac{1}{x^3} && \text{(See Section 1.5)} \\
(x+a)^2 &\neq x^2 + a^2 && \text{(See online resources for proof)} \\
2(x+1)^2 &\neq (2x+2)^2 && \text{(See online resources for proof)}
\end{aligned}$$

For further algebraic tools you feel you might be missing, read up on them here: `http://tutorial.math.lamar.edu/pdf/Algebra_Cheat_Sheet.pdf`

For further assistance with trigonometry vist: `http://tutorial.math.lamar.edu/pdf/Trig_Cheat_Sheet.pdf`